Math 10A with Professor Stankova
Quiz 2; Wednesday, 9/6/2017
Section #106; Time: 10 AM
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Circle True or False or leave blank. (1 point for correct answer, −1 for incorrect answer, 0 if left blank)

1. **TRUE** False The range of an invertible function f is the domain of the inverse f^{-1}.
2. True **FALSE** If a function f is defined at $x = 0$, then we must have $\lim_{x \to 0} f(x) = f(0)$.

Show your work and justify your answers.

3. (10 points) Let $f(t) = \dfrac{\sqrt{9 - t^2} - 3}{t^2}$.

   (a) (2 points) What is the domain of f?

   Solution: The domain is when $9 - t^2 \geq 0$ and when $t^2 \neq 0$. So $D = \{t : t^2 \leq 9\} \cap \{t : t \neq 0\} = [-3, 0) \cup (0, 3] = \{t : -3 \leq t \leq 3 \text{ and } t \neq 0\}$.

   (b) (3 points) Find $\lim_{t \to \sqrt{5}} f(t)$.

   Solution: Since f is continuous at $t = \sqrt{5}$, which we know since it is a combination of polynomials, the limit is just $f(\sqrt{5}) = \dfrac{\sqrt{9 - \sqrt{5}^2} - 3}{\sqrt{5}^2} = \dfrac{\sqrt{4} - 3}{5} = \dfrac{-1}{5}$.

   (c) (5 points) Find $\lim_{t \to 0} f(t)$.

   Solution: We have that

$$\lim_{t \to 0} \frac{\sqrt{9 - t^2} - 3}{t^2} = \lim_{t \to 0} \frac{\sqrt{9 - t^2} - 3}{t^2} \cdot \frac{\sqrt{9 - t^2} + 3}{\sqrt{9 - t^2} + 3}$$

$$= \lim_{t \to 0} \frac{9 - t^2 - 9}{t^2(\sqrt{9 - t^2} + 3)} = \lim_{t \to 0} \frac{-1}{\sqrt{9 - t^2} + 3} = \frac{-1}{\sqrt{9} + 3} = \frac{-1}{6}.$$